Double and iterated integrals

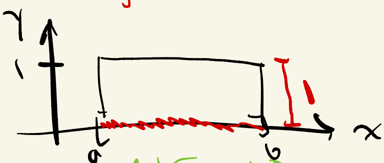
- compute volume under surface
  (single var: area under curve)

Recall: MAT 21B

height of shape

$$\int_{\text{line seg}} 1 \, dx = \int_{[a,b]} 1 \cdot dx = \int_a^b 1 \cdot dx = (b-a) \cdot 1$$

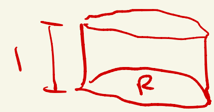
size of integration region (domain)



By integrating the function 1, we computed the "size" of our integration region

MAT 21D

$$\boxed{\iint_R 1 \cdot dA = \text{Area}(R)}$$

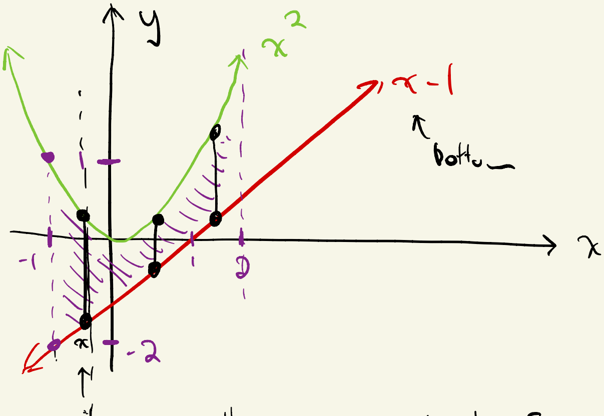


15.2:

Sketch regions of integration and calculate area

(2) R $\quad -1 \le x \le 2$

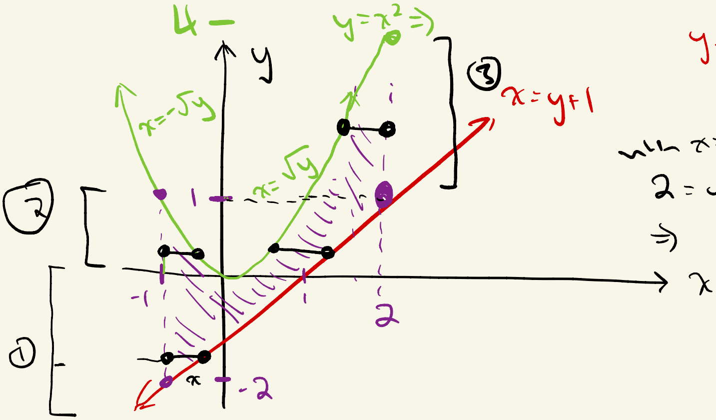
$$x-1 \le y \le x^2$$



- Fix x. How can we think of y as a function of x?

$$\text{Area} = \int_{x=-1}^{x=2}\int_{y=x-1}^{y=x^2} 1 \cdot dy\,dx = \int_{-1}^{2}\int_{x-1}^{x^2} dy\,dx$$

$$= \frac{9}{2}$$



$y=x-1$

when $x=2$,
$2=y+1$
$\Rightarrow \quad y=1$

Fix y. How is x a function of y?

① $\displaystyle\int_{y=-2}^{y=0}\int_{x=-1}^{x=y+1} dx\,dy$ $\quad 2$

$+$

② $\displaystyle\int_{y=0}^{y=1}\int_{x=-1}^{x=-\sqrt{y}} dx\,dy + \int_{y=0}^{y=1}\int_{x=\sqrt{y}}^{x=y+1} dx\,dy$ $\quad \frac{1}{3}+\frac{5}{6}$

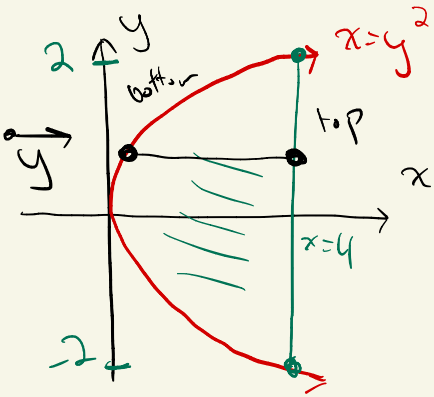
$+$

③ $\displaystyle\int_{y=1}^{y=4}\int_{x=\sqrt{y}}^{x=2} dx\,dy$ $\quad \frac{4}{3}$

$= \frac{9}{2}$

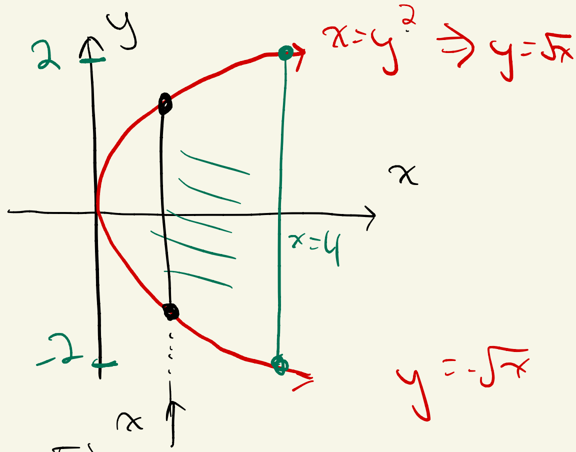
Moral: choose functions wisely

③ R $\quad -2 \le y \le 2, \quad y^2 \le x \le 4$



Fix y.

$$\int_{y=-2}^{y=2}\int_{x=y^2}^{x=4} 1 \cdot dx\, dy = \frac{32}{3}$$



Fix x.

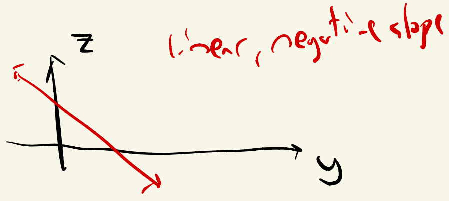
$$\int_{x=0}^{x=4}\int_{y=-\sqrt{x}}^{y=\sqrt{x}} 1 \cdot dy\, dx = \frac{32}{3}$$

(6) Volume of solid cut from the first octant bounded by surface $z = 4 - x^2 - y$.

How to think about this function?

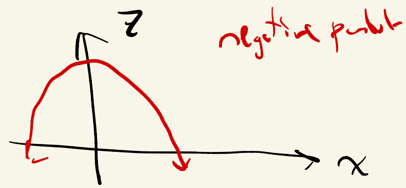
- well, if x is fixed, then

$$z = \underbrace{\quad\quad}_{\substack{\text{constant} \\ \text{(depends on } x)}} - y$$



- And if y is fixed, then

$$z = -x^2 + \underbrace{\quad\quad}_{\substack{\text{constant} \\ \text{(depending on } y)}}$$



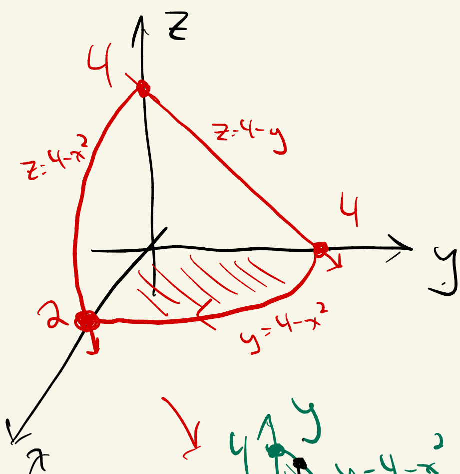
at $(x, y) = (0, 0)$, $z = 4$.

(yz plane)
- $x = 0 \Rightarrow z = 4 - y$

(xz plane)
- $y = 0 \Rightarrow z = 4 - x^2$

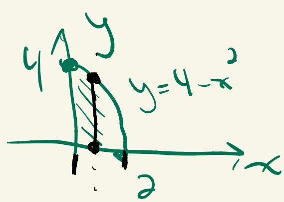
(xy plane)
- $z = 0 \Rightarrow 0 = 4 - x^2 - y$

$\Rightarrow y = 4 - x^2$



$$\text{Volume} = \int_{x=0}^{x=2} \int_{y=0}^{y=4-x^2} 4 - x^2 - y \; dy\, dx$$

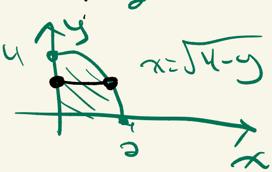
$$\frac{16}{3} = \int_{y=0}^{y=4} \int_{x=0}^{x=\sqrt{4-y}} 4 - x^2 - y \; dx\, dy$$



$y = 4 - x^2$

$x^2 = 4 - y$

$x = \sqrt{4-y}$ (positive branch only)



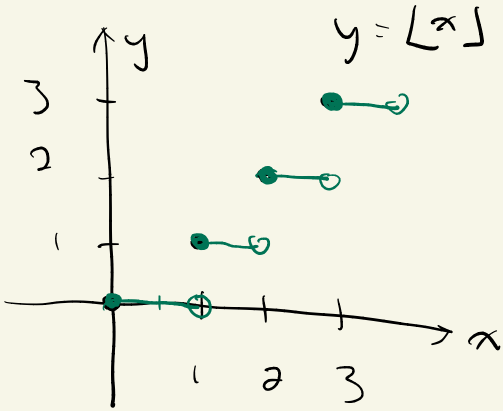
PS 1 # 2 (Hints):

Floor Function = round number down

$\lfloor 1.5 \rfloor = 1$

$\lfloor \pi \rfloor = 3$

$\lfloor -2.5 \rfloor = -3$



<u>Not</u> continuous

(is piecewise continuous

PS 1 # 3

Recall: (21B) Fundamental Thm calculus

$$\int_a^b \frac{dF}{dx}(x)\,dx = F(b) - F(a)$$

where F is a (differentiable) function of x on $[a,b]$

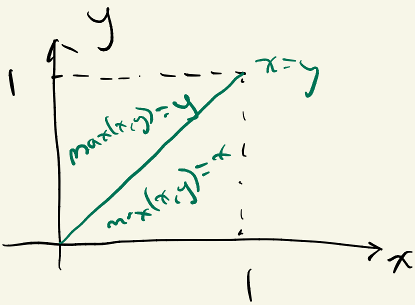
$F(x,y)$ For any y, $F(x,y)$ is a function of x

$$\int_a^b \frac{\partial F}{\partial x}(x,y)\,dx = F(b,x) - F(a,x)$$

PS 1 #6

$e^{(\max(x,y))}$

$$\max(x,y) = \begin{cases} x & x \geq y \\ y & x < y \end{cases}$$



Flips when $x=y$

Survey due at 11:40